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Nongeometric flux compactifications

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Abstract: We investigate a simple class of type-II string compactifications which incorporate nongeometric "fluxes" in addition to "geometric flux" and the usual H-field and R-R fluxes. These compactifications are nongeometric analogues of the twisted torus. We develop T-duality rules for NS-NS geometric and nongeometric fluxes, which we use to construct a superpotential for the dimensionally reduced four-dimensional theory. The resulting structure is invariant under T-duality, so that the distribution of vacua in the IIA and IIB theories is identical when nongeometric fluxes are included. This gives a concrete framework in which to investigate the possibility that generic string compactifications may be nongeometric in any duality frame. The framework developed in this paper also provides some concrete hints for how mirror symmetry can be generalized to compactifications with arbitrary H-flux, whose mirrors are generically nongeometric.

Keywords: String Duality, Flux compactifications, Superstring Vacua.

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1. Introduction

Since the early days of string theory, it has been clear that there are many possible ways in which to compactify the various perturbative superstring and supergravity theories from ten or eleven dimensions to four space-time dimensions. For example, compactifying any ten-dimensional string theory on a Calabi-Yau complex three-fold leads to a supersymmetric theory of gravity coupled to light fields in the remaining four macroscopic space-time dimensions. Moduli parameterizing the size and shape of the Calabi-Yau appear as massless scalar fields in the four-dimensional theory. Understanding and classifying the range of possible compactifications is an important part of the program of relating superstring theory to observed phenomenology and cosmology. In recent years, compactifications with topologically quantized fluxes wrapping compact cycles on the compactification manifold have become a subject of much interest, following the work of [1 − 3]. The topological fluxes produce a potential for the scalar moduli, and can thus "stabilize" some or all of the moduli to take specific values [4 − 6]. Once fluxes are added to the system, however, the geometric structure of the compactification manifold may also become more general. Recent work has addressed the generalization to superstring compactifications on non-Calabi-Yau geometries [7 − 10].

The goal of the present work is to take the study of flux compactifications one step further, by including "compactifications" which cannot be described by a geometric ten-dimensional space-time manifold. It was argued in [11] that nongeometric flux compactifications arise naturally as configurations which are T-dual to known geometric supersymmetric flux compactifications. To be specific, consider for example a compactification on a six-torus T^6 with NS-NS 3-form flux H_{abc} on some three-cycle, where indices $a, b, \ldots \in \{1, 2, \ldots, 6\}$ take values in the compact directions. Under a single T-duality, say in direction a, this H-flux is mapped to "geometric flux" associated with a twist in the torus topology. In the presence of this geometric flux, the metric on the twisted torus acquires a contribution which can be written as $(dx^a - f^a_{bc} x^c dx^b)^2$, where f^a_{bc} is integrally quantized and characterizes the "geometric flux" of the compactification. This kind of "twisted torus" has been studied as a type of Scherk-Schwarz compactification for many years [12 – 14], and is considered in the context of flux compactification in [8, 15 – 20], among others. Even in the presence of geometric flux f^a_{bc}, however, as was pointed out in [11], we can perform another T-duality on direction b, since the metric can be chosen to be independent of the coordinate x^b. Carrying out this T-duality explicitly leads to a dual "torus", which is locally geometric, but which cannot be described globally in terms of a fixed geometry, due to the appearance of a nongeometric duality transformation in the boundary conditions which patch together local descriptions of the compactification space. Nongeometric spaces of this type were considered in [21 – 25]. In this paper we label the nongeometric flux resulting from T-duality T_b by Q^{ab}_c, and we determine how fluxes Q^{ab}_c can be incorporated in the superpotential for a simple T-duality invariant class of IIA and IIB compactifications. Note that although we use T-duality to determine the role of the fluxes in the superpotential, a generic configuration with fluxes H, f, Q turned on cannot be T-dualized to a completely geometric compactification.

After performing the second T-duality to a configuration with nongeometric flux Q^{ab}_c, there is no apparent residual isometry around direction c. Naïvely it does not seem that a further T-duality can be performed. Nonetheless, we find a structure suggesting that there is some meaning which can be given to a T-dual flux of this type, which we label R^{abc}. While we do not have an explicit presentation of a nongeometric compactification with such nongeometric fluxes, it seems that this structure should have some meaning in any background-independent formulation of the theory. The situation here is analogous to that for R-R fluxes. The Buscher rules [26 – 28] for T-duality act on the p-form R-R fields $A^{(p)}$, and therefore cannot be used to explicitly construct configurations with R-R flux $F^{(0)}$, just as they cannot be used to construct R^{abc}, which acts formally as an NS-NS 0-form flux. In the case of $F^{(0)}$, it is necessary to use T-duality rules which act directly on the R-R fluxes [29, 30, 5] to map $F^{(1)} \to F^{(0)}$. Acting on the integrated R-R fluxes, these T-duality rules take

$$F_{x\alpha_1\cdots\alpha_p} \stackrel{T_x}{\longleftrightarrow} F_{\alpha_1\cdots\alpha_p}\,. \tag{1.1}$$

The T-duality rules we construct in this paper for nongeometric fluxes, which take

$$H_{abc} \stackrel{T_a}{\longleftrightarrow} f^a_{bc} \stackrel{T_b}{\longleftrightarrow} Q^{ab}_c \stackrel{T_c}{\longleftrightarrow} R^{abc}\,, \tag{1.2}$$

can be thought of as an extension of (1.1) to a general class of integral NS-NS fluxes. The structure we find here for the simple toroidal example suggests that in addition to H-form flux, both geometric and nongeometric fluxes can be thought of as additional algebraic structure added to a given string background. In the geometric context, this kind of structure for geometric fluxes as data which decorate a Calabi-Yau seems to arise naturally when the mirror of a Calabi-Yau with H-flux has a geometric description [8, 9]. Another perspective on the geometric and nongeometric structures arising from the T-dualities in (1.2) is given in [31].

The approach taken in this paper is as follows: in section 2, we incorporate nongeometric fluxes by using T-duality and coordinate transformations to construct the complete set of terms which may appear in the superpotential for a class of compactifications based on a symmetric $T^6 = (T^2)^3$ with all T^2 components identical. The resulting simple polynomial superpotential subsumes the previously known superpotentials for geometric compactifications of the IIA and IIB theories, and extends them to a T-duality invariant form which includes both nongeometric Q^{ab}_c and R^{abc} types of "fluxes." In section 3 we give a more detailed discussion of the interpretation of Q^{ab}_c and R^{abc} fluxes, and provide an explicit description of "T-folds" [24] with Q-structure. In section 4 we discuss constraints on the fluxes, which arise both from tadpole cancellation requirements in the presence of orientifolds, and from Bianchi-type identities. Finally, in section 5 we conclude and discuss some directions for future research.

2. Nongeometric fluxes and the superpotential

In this section we use various dualities to explicitly construct a simple low-energy effective theory governing a class of geometric and nongeometric compactifications. We begin by considering the compactification of type-IIA and type-IIB string theory on a torus $T^6 = (T^2)^3$, where each T^2 factor represents an identical torus. We can think of this as a compactification on a T^6 with additional discrete symmetries imposed. We impose a symmetry under $\mathbb{Z}_2$ which reflects the first two 2-tori under $(-1, -1, -1, -1, 1, 1)$ and a further symmetry under a $\mathbb{Z}_3$ which rotates the tori $T^2_{(1)} \to T^2_{(2)} \to T^2_{(3)} \to T^2_{(1)}$. In the IIB theory we then have a single complex structure modulus τ parameterizing the complex structure of the T^2, a single Kähler modulus U containing the C_4 modulus and the scale of the T^2 and an axiodilaton S. In the IIA theory which arises after 3 T-dualities (one on each T^2), τ becomes the Kähler modulus and U becomes the complex structure modulus [19, 20]. Flux compactifications of this type were considered in type-IIB in [5, 6, 32, 33] and in IIA in [19, 20]. A slightly more general model was also considered in these papers, where the three complex structure and Kähler moduli are allowed to vary independently by imposing a second $\mathbb{Z}_2$ symmetry $(1, 1, -1, -1, -1, -1)$ instead of the $\mathbb{Z}_3$ we use here. This model can be seen as a special case of that $T^6/\mathbb{Z}_2^2$ model. It is straightforward to generalize the considerations here to that more general model, with slightly more algebra.

We wish to include various kinds of fluxes on the T^6. When an orientifold is included to cancel tadpoles, these fluxes lead generally to a low-energy effective $\mathcal{N} = 1$ supergravity theory in four dimensions which has a superpotential $W(\tau, U, S)$, a Kähler potential $K(\tau, U, S)$, and a resulting potential for the moduli given by

$$V = e^K \left(\sum_{i,j,=\{\tau,U,S\}} K^{ij} D_i W \overline{D_j W} - 3|W|^2 \right), \tag{2.1}$$

where K^{ij} is the inverse of $K_{ij} = \partial_i \bar{\partial}_j K$. The construction of flux compactifications with orientifolds was developed in [2, 3], and applied to the IIB theory in [4] and many subsequent papers, and to the IIA theory in [16, 19, 20, 34–36].

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An important caveat which must be taken into account when describing flux compactifications through the dimensionally reduced four-dimensional theory is that the low-energy four-dimensional supergravity action is only valid when the moduli acquire masses which are small compared to those of fields such as higher string modes, winding modes, and Kaluza-Klein modes which are neglected in the dimensional reduction. This issue must be addressed in any study of flux compactifications. In the class of vacua we consider here, which are not geometric, and for which ten-dimensional supergravity is not a valid approximation, this question becomes even more subtle. For the present, we will simply describe the superpotential for the four-dimensional supergravity theory as a function of the degrees of freedom associated with the original moduli on the torus. This allows us at least to characterize the topological features of the nongeometric fluxes in which we are interested. We leave a more detailed study of the regime of validity of the low-energy theory in the presence of nongeometric fluxes to further work.

The particular symmetric torus $T^6 = (T^2)^3$ model we are interested in here was studied in [5, 32] and explicitly solved in [33] for IIB compactifications with R-R and NS-NS form field flux. In this case the superpotential is given by

$$W_{\rm IIB} = P_1(\tau) + SP_2(\tau) \tag{2.2}$$

where $P_{1,2}(\tau)$ are cubic polynomials in τ. The Kähler potential is

$$K = -3\ln(-i(\tau-\bar{\tau})) - \ln(-i(S-\bar{S})) - 3\ln(-i(U-\bar{U}))\,. \tag{2.3}$$

In [19, 20] this model was studied for the IIA theory, where in addition to NS-NS and R-R form field fluxes, geometric flux was also allowed. In this case the Kähler potential is again (2.3), while the superpotential is

$$W_{\rm IIA} = P_1(\tau) + SP_2(\tau) + UP_3(\tau)\,, \tag{2.4}$$

with P_1 again cubic, but with $P_{2,3}$ now linear in τ.

In order to consider a complete T-duality invariant family of flux compactifications we must extend somewhat the nature of allowed fluxes. As discussed in the Introduction, we must include not only geometric fluxes but also some structures we interpret as "nongeometric fluxes". Simply using T-duality and coordinate symmetries, we can proceed to construct the full duality-invariant superpotential W, identifying the fluxes corresponding to each term in W. We now proceed to directly present this superpotential, which is one of the main results of this paper, after which we give a more detailed discussion of how the various terms are derived through dualities. The later sections of the paper discuss the interpretation of the fluxes which appear in this potential, and constraints on these quantized fluxes.

We claim that the full potential for the symmetric torus in both the IIA and IIB theories is given by

$$W_{\rm complete} = P_1(\tau) + SP_2(\tau) + UP_3(\tau)\,, \tag{2.5}$$

Term	IIA flux	IIB flux	integer flux
1	$\bar{F}_{\alpha i\beta j\gamma k}$	$\bar{F}_{ijk}$	a_0
τ	$\bar{F}_{\alpha i\beta j}$	$\bar{F}_{ij\gamma}$	a_1
τ^2	$\bar{F}_{\alpha i}$	$\bar{F}_{i\beta\gamma}$	a_2
τ^3	$F^{(0)}$	$\bar{F}_{\alpha\beta\gamma}$	a_3
S	$\bar{H}_{ijk}$	$\bar{H}_{ijk}$	b_0
U	$\bar{H}_{\alpha\beta k}$	$Q_k^{\alpha\beta}$	c_0
$S\tau$	f_{jk}^{α}	$\bar{H}_{\alpha jk}$	b_1
$U\tau$	$f_{k\alpha}^{j}, f_{\beta k}^{i}, f_{\beta\gamma}^{\alpha}$	$Q_k^{\alpha j}, Q_k^{i\beta}, Q_\alpha^{\beta\gamma}$	$\check{c}_1, \hat{c}_1, \tilde{c}_1$
$S\tau^2$	$Q_k^{\alpha\beta}$	$\bar{H}_{i\beta\gamma}$	b_2
$U\tau^2$	$Q_\beta^{\gamma i}, Q_\gamma^{i\beta}, Q_k^{ij}$	$Q_\gamma^{i\beta}, Q_\beta^{\gamma i}, Q_k^{ij}$	$\check{c}_2, \hat{c}_2, \tilde{c}_2$
$S\tau^3$	$R^{\alpha\beta\gamma}$	$\bar{H}_{\alpha\beta\gamma}$	b_3
$U\tau^3$	$R^{ij\gamma}$	Q_γ^{ij}	c_3

Table 1: Fluxes appearing as coefficients of terms in the superpotential.

where now all three of $P_{1,2,3}(\tau)$ are cubic polynomials. The coefficients in these polynomials are given in the IIB theory by (integrally quantized) NS-NS and R-R fluxes $\bar{H}_{abc}, \bar{F}_{abc}$ (denoting the integral number of units of flux of, e.g., F_{abc} by $\bar{F}_{abc}$) and also by "nongeometric" fluxes Q_c^{ab} (which each can individually arise as the T-dual on direction b of the geometric flux f_{bc}^{a}). In the IIA theory, the coefficients include (integrally quantized) R-R p-form fluxes $F^{(0)}, \bar{F}_{ab}, \bar{F}_{abcd}, \bar{F}_{abcdef}$, as well as NS-NS 3-form flux $\bar{H}_{abc}$, geometric fluxes f_{bc}^{a}, nongeometric fluxes Q_c^{ab}, and further nongeometric fluxes R^{abc} (which can individually be seen formally as the T-dual on c of Q_c^{ab}). In the next section we discuss the interpretation of these nongeometric fluxes in more detail. For now, however, we will simply show how dualities determine which fluxes arise as which coefficients in the superpotential (2.5).

To make the discussion more explicit, we label coordinates $1, 3, 5$ on the T^6 with indices α, β, γ and coordinates $2, 4, 6$ with indices i, j, k. The IIB torus is taken to have an O3-plane filling four-dimensional space-time, so that all internal coordinates are odd under the orientifold reflection Ω. To get to the IIA theory we T-dualize on the dimensions $1, 3, 5$, in that order, so that the resulting IIA O6-plane extends along these dimensions with indices α, β, γ. In the table 1, we list the fluxes associated with each term in the superpotential (2.5) in both IIA and IIB.

To be explicit about the index structure in this table, notice that the orbifold projection we have chosen implies that all objects with three indices must have one index on each T^2; our convention in the table 1 is that these indices are ordered cyclically by T^2 in the fashion indicated by the Greek and Latin indices; thus for example $Q_k^{\alpha\beta} = Q_6^{13} = Q_2^{35} = Q_4^{51}$. The IIA R-R forms that survive the orbifold projection must have pairs of indices extending on both dimensions of each T^2 on which there are any indices. We denote here by $\alpha i, \beta j, \gamma k$ pairs of indices on the same torus; thus, for example $\bar{F}_{\alpha i} = \bar{F}_{12} = \bar{F}_{34} = \bar{F}_{56}$. Additionally, all fluxes in the table 1 are antisymmetric in their upper indices as well as in their lower indices. Note that while f and R fluxes do not appear on the IIB side of the table 1, this is a consequence of the fact that all dimensions on the T^6 are odd under the orientifold reflection, and f and R require an even number of odd indices. In more general orientifolds, IIB compactifications would also admit f and R fluxes.

The resulting full superpotential in the symmetric torus model is

$$
\begin{aligned}
W = & \, a_0 - 3a_1\tau + 3a_2\tau^2 - a_3\tau^3 + S(-b_0 + 3b_1\tau - 3b_2\tau^2 + b_3\tau^3) + \\
& + 3U(c_0 + (\hat{c}_1 + \check{c}_1 - \tilde{c}_1)\tau - (\hat{c}_2 + \check{c}_2 - \tilde{c}_2)\tau^2 - c_3\tau^3)\,. \qquad (2.6)
\end{aligned}
$$

At this point let us comment briefly on the nature of the integrally quantized fluxes appearing in the table 1. On the standard geometric torus, by $\bar{H}_{ijk}, \bar{F}_{\alpha i}, \ldots$ we simply mean the number of units of H or F integrated over the appropriate cycle on the torus. In the presence of geometric flux such as f^a_{bc} this is slightly more subtle, but can still be made explicit. There is a natural basis of Einbeins η^a satisfying $d\eta^a = f^a_{bc}\eta^b \wedge \eta^c$ [11], which we may use to obtain integrally quantized fluxes such as $F^{(2)} = \bar{F}_{ab}\eta^a \wedge \eta^b$ for any pair a, b, even when the corresponding R-R flux is not in the cohomology of the manifold. For example, turning on units of flux $\bar{H}_{123} = 1, \bar{F}_4 = M$ on a standard 4-torus gives a configuration which is taken by T-duality to $f^1_{23} = 1, \bar{F}_{14} = M$ on the dual torus with geometric flux. While the resulting R-R flux $F^{(2)}$ is not in the cohomology (since the 1-cycle is trivial in homotopy), there is still a nontrivial integral quantization, as we see from this explicit T-duality. There are constraints due to tadpole cancellation and integrated Bianchi identities which we discuss in section 4; these place linear constraints on the R-R fluxes in a fixed geometric background with nonzero f^a_{bc}. This is presumably how the K-theory description of R-R charges [37] continues to be valid in the case with geometrical fluxes (possibly related considerations appeared in [38]). We do not have a complete understanding of how this works in detail, however, particularly when there is torsion in the cohomology. In the cases with nongeometric fluxes Q^{ab}_c and R^{abc}, we do not have a specific and concrete interpretation of the meaning of the quantized fluxes, but from the approach we take here it seems natural to associate integral fluxes $\bar{H}_{abc}, \bar{F}_{a_1 \cdots a_p}$ with every cycle on the original torus; these fluxes appear in the superpotential and are constrained by the identities we compute in section 4. These fluxes are perhaps best interpreted as some "dressing" added to the basic topological structure of the geometrical torus, in a way which might naturally generalize to other Calabi-Yau manifolds.

Let us now discuss the detailed derivation of the arrangement of fluxes in the table above. As discussed above, the IIB NS-NS and R-R fluxes appearing in P_1 and P_2 are already known [2, 3, 5], as are the IIA NS-NS, R-R, and geometric fluxes appearing in P_1 and the linear parts of P_2 and P_3 [16, 19, 20]. We need to complete the story using T-duality and coordinate symmetry. Our conventions for the action of T-duality on topological R-R and NS-NS fluxes, including the nongeometric fluxes, is that T-duality removes or adds an index to the first position of an R-R flux, $T_x : \bar{F}_{i_1 \cdots i_n} \leftrightarrow \bar{F}_{x i_1 \cdots i_n}$, and acts on NS-NS fluxes by either raising the first lower index or lowering the last upper index, so for example $T_b : f^a_{bc} \leftrightarrow Q^{ab}_c$. Since we are only interested in the action of T-duality on the topological part of the fluxes, additional moduli-dependent terms which appear in the local T-duality rules [26, 27, 29, 30, 5] are not relevant to this discussion.

We begin by noting that the first eight lines in the table all contain known fluxes in the IIA picture. Each of these fluxes can be T-dualized directly to IIB. The R-R fluxes transform to the known IIB R-R flux coefficients. The S and $S\tau$ terms are associated with NS-NS H-fluxes and geometric fluxes which transform to the known H-fluxes in the IIB theory. The U and $U\tau$ terms can also be T-dualized, and lead to new coefficients in the IIB theory associated with nongeometric fluxes Q_c^{ab} for various values of a, b, c. Thus, we can use T-duality to complete the picture for the first 8 lines in the table.

To proceed further we note that in the IIB model, there is no geometric distinction between α and i indices, as the O3-plane does not extend in any of the directions on the T^6. Thus, by switching the roles of the α and i indices in defining the complex structure, we exchange $\alpha \leftrightarrow i$, etc.. This exchange takes $1 \leftrightarrow \tau^3, \tau \leftrightarrow \tau^2$ in the superpotential, and allows us to identify the remaining Q_c^{ab} coefficients associated with $U\tau^2, U\tau^3$ in the IIB superpotential.

This derivation relies only on duality transformations which can be performed explicitly (at least for each individual flux), and therefore is a rigorous demonstration that the IIB theory has the full superpotential (2.5) with all coefficients in the cubic polynomials $P_{1,2,3}(\tau)$ associated with well-defined geometric and nongeometric fluxes. A more detailed discussion of the nongeometric fluxes Q_c^{ab} appears in the following section.

To complete the story on the IIA side, we would like to carry our results from IIB back to IIA using the 3-fold T-duality on the complete IIB superpotential. The $S\tau^2$ and $U\tau^2$ terms are associated in the IIB theory with fluxes which transform to nongeometric fluxes Q_c^{ab} on the IIA side. This extends the superpotential constructed in [19, 20] to include nongeometric fluxes of the Q-form. Note, however, that the terms $S\tau^3$ and $U\tau^3$ are associated with fluxes in the IIB theory which in the IIA theory must take the form of a "T-dual" on direction c of a nongeometric flux Q_c^{ab}. We do not know how to carry out such a transformation explicitly. It seems, however, that for duality invariance to be complete, we must introduce a new type of nongeometric flux in the IIA theory, labeled R^{abc}. We discuss the possible interpretation of these new topological nongeometric fluxes in the next section. Including these fluxes leads to a superpotential (2.5) which is manifestly T-duality invariant. Note that with this complete set of fluxes, not only can we go from the IIB theory with an O3-plane to the IIA theory with an O6-plane, but we can also perform a complete 6-fold duality on the IIA theory. This duality flips the complex structure on each T^2, again taking $1 \leftrightarrow \tau^3, \tau \leftrightarrow \tau^2$. Again, for this to be an invariance of W we must include the fluxes $R^{\alpha\beta\gamma}, R^{ij\gamma}$, which in this case are the 6-fold T-duals of $\bar{H}_{ijk}, \bar{H}_{\alpha\beta k}$.

This completes our construction of the duality-invariant superpotential for orientifold compactifications of the generalization of the twisted torus in type-IIA and IIB string theory. As we have seen, nongeometric fluxes appear as coefficients of various terms in this superpotential. In the following sections we will discuss the interpretation of these nongeometric fluxes and topological constraints on possible values of these fluxes.

Given the superpotential we have computed here, it is straightforward in principle to choose integral fluxes $a_0, \ldots$ (subject to constraints which we will discuss in section 4) and to solve the equations of motion. Given the superpotential (2.5) and the Kähler poten-

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tial (2.3), the equations for a supersymmetric vacuum in the four-dimensional theory are

$$D_\tau W = D_S W = D_U W = 0\,, \tag{2.7}$$

where

$$D_A W = \partial_A W + (\partial_A K) W\,. \tag{2.8}$$

For generic flux coefficients in the superpotential (2.5), the equations for S and U are equivalent to

$$P_1(\tau) + \bar{S} P_2(\tau) + U P_3(\tau) = 0 \tag{2.9}$$

$$P_1(\tau) + S P_2(\tau) + \left(\frac{2}{3}U + \frac{1}{3}\bar{U}\right) P_3(\tau) = 0\,. \tag{2.10}$$

The remaining (τ) equation is

$$(\tau - \bar{\tau})\partial_\tau W - 3W = 0\,. \tag{2.11}$$

We defer a detailed analysis of solutions of these equations to a forthcoming paper [39], but we will make a few brief comments here regarding the space of solutions. Clearly, the space of SUSY solutions to these equations will include all type-IIB and IIA flux vacua on the geometric symmetric torus, as well as possibly a large number of vacua with geometric and nongeometric fluxes, which may generically have no geometric duals. In [33], a family of supersymmetric IIB vacua on the symmetric torus with $W = 0$ was identified, corresponding to flux compactifications with vanishing cosmological constant. These vacua all have nonvanishing b_2 or b_3 and therefore are nongeometric in the IIA picture. Indeed, it is easy to see from (2.9)–(2.11) that there are no geometric $W = 0$ solutions in the IIA theory, since this would require either Im $S = 0$ (which is unphysical) or $P_2 = 0$. The vanishing of P_2 in turn implies that either $b_0 = b_1 = 0$ (which violates the tadpole condition (4.9), which we derive in general in section 4), or $\tau = b_0/3b_1$ is real (which is again unphysical). Thus, admitting nongeometric fluxes expands the set of Minkowski IIA flux vacua on the symmetric torus from the empty set to a nonzero set of vacua. In [20] it was argued that when geometrical fluxes are allowed, there are an infinite family of AdS vacua in the IIA torus model. Assuming this result is correct, this shows that including nongeometric fluxes on the IIB side extends the finite set of geometric SUSY Minkowski vacua by an infinite set of AdS vacua. Beyond these already known results, it seems that generic nongeometric flux configurations may have no geometric duals, but may nevertheless lead to acceptable SUSY flux compactifications. A more detailed analysis of this issue will appear in [39].

3. Interpretation of nongeometric fluxes

In this section we describe in greater detail the structure of “T-folds” with nongeometric Q-fluxes, and speculate about the nature of R-fluxes. So far, our treatment of these fluxes has been fairly formal. We have essentially defined a set of T-duality transformation rules for generalized NS-NS fluxes on the torus through (1.2), analogous to the T-duality

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rules for R-R fluxes. In this section we discuss the nongeometric fluxes Q_c^{ab} and R^{abc} in greater detail, and discuss when compactifications with such extra structure can be described at least locally geometrically. We also comment on the world-sheet description of compactifications with nongeometric fluxes.

In order to develop some intuition for the nongeometric fluxes Q_c^{ab}, let us discuss a simple example where only the flux Q_c^{ab} is present. We will construct this configuration by step by step using T-duality, as in [11, 15], beginning from a square three-torus with metric

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{3.1}$$

and N units of H-flux, $\bar{H}_{xyz} = N$. We are free to choose a gauge where

$$B_{xy} = Nz\,. \tag{3.2}$$

We can think of this configuration as a T^2 parameterized by x and y, fibered over a circle with coordinate z. The NS-NS degrees of freedom coming from reduction on T^2_{xy} are the complex structure modulus of the torus, τ, and the Kähler modulus $\rho = B_{xy} + i\mathrm{vol}_{xy}$. The perturbative duality group of the theory reduced on T^2_{xy} is, up to discrete factors, $SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho$. The presence of N units of H-flux is described in this language as a nontrivial monodromy $\rho \to \rho + N$ as $z \to z+1$.

Since it will be useful in our discussion of the nongeometric fluxes, let us take a moment to develop this $\mathrm{SL}(2,\mathbb{Z})$ description of compactification with H-flux a little further. This will help in understanding a general set of quadratic constraints on the fluxes which we derive in the next section. In an ordinary dimensional reduction, we take the fields to be independent of the coordinates of the compact directions. In a reduction with H-flux, there is nontrivial coordinate dependence in the gauge potentials. Dimensional reduction in the presence of flux can thus be understood as a class of Scherk-Schwarz generalized dimensional reduction [12, 14]. From this point of view, the z-dependence of the fields is understood by specifying an z-dependent element $M(z) \in \mathrm{SL}(2,\mathbb{R})_\rho$ which has the desired monodromy $\left(\begin{smallmatrix} 1 & N \\ 0 & 1 \end{smallmatrix}\right)$ when $z \to z+1$ [40, 41]. The theory reduced on the three-torus must be independent of z, so in the present example $M(z)$ can be at most linear in z. Our choice of gauge (3.2) for B gives us

$$M(z) = \begin{pmatrix} 1 & Nz \\ 0 & 1 \end{pmatrix}_\rho ; \tag{3.3}$$

other choices of $M(z)$ with the same monodromy are possible and lead to reduced theories which are equivalent under field redefinitions [41]. The reduction ansatz for an arbitary field ϕ is then $\phi(z) = [M(z)]_\phi\, \phi_0$, where $[M(z)]_\phi$ is the appropriate representation of $M(z)$, and ϕ_0 is a vector in this representation containing the degrees of freedom analogous to zero-modes in this background.

Let us consider the case where the degrees of freedom are R-R field strengths; this will be useful in understanding how turning on topological NS-NS fluxes affects the topological R-R fluxes. For illustration, consider the topological fluxes $\bar{F}_{wxy}$ and $\bar{F}_w$ in type-IIB. Here

w denotes some compact direction transverse to the three-torus. These degrees of freedom transform under $\mathrm{SL}(2,\mathbb{Z})_\rho$ as

$$\begin{pmatrix} \bar{F}_{wxy} \\ \bar{F}_w \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \bar{F}_{wxy} \\ \bar{F}_w \end{pmatrix}. \tag{3.4}$$

Therefore, using the matrix (3.3) to describe the presence of H-flux gives us

$$\begin{pmatrix} F_{wxy}(z) \\ F_w(z) \end{pmatrix} = \begin{pmatrix} 1 & Nz \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{F}_{wxy} \\ \bar{F}_w \end{pmatrix}, \tag{3.5}$$

so

$$F^{(3)} = (\bar{F}_{wxy} + Nz\,\bar{F}_w)dw\,dx\,dy\,, \tag{3.6}$$

from which we can recover the familiar Bianchi identity

$$dF^{(3)} = -N\bar{F}_w\,dw\,dx\,dy\,dz = -\bar{F}^{(1)} \wedge \bar{H}\,. \tag{3.7}$$

As usual, integrating this equation on the (w,x,y,z)-cycle leads to the constraint

$$\bar{F}_{[w}\bar{H}_{xyz]} = 0\,. \tag{3.8}$$

Note that when other fluxes are present, including geometric and nongeometric fluxes, this constraint will acquire other terms. The point of view we used to obtain this constraint will prove very useful in obtaining the analogous constraint terms in the presence of geometric flux and nongeometric Q-flux, as we will demonstrate below.

Following [11], we take this square three-torus with H-flux and perform a T-duality on the x direction. This yields a twisted T^3 with $f^x_{yz} = N$, which in this gauge has the metric

$$ds^2 = (dx - Nzdy)^2 + dy^2 + dz^2 \tag{3.9}$$

and $B = 0$. The nontrivial monodromy is now $\tau \to \tau - N$, realized by the action of the matrix

$$M(z) = \begin{pmatrix} 1 & -Nz \\ 0 & 1 \end{pmatrix}_\tau \tag{3.10}$$

on the fields in the theory.

Consider now the behavior of Ramond-Ramond fluxes in this background. For concreteness, consider the fluxes $\bar{F}_{wx}$ and $\bar{F}_{wy}$ in type-IIA. These transform under $\mathrm{SL}(2,\mathbb{Z})_\tau$ as

$$\begin{pmatrix} \bar{F}_{wy} \\ \bar{F}_{wx} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \bar{F}_{wy} \\ \bar{F}_{wx} \end{pmatrix}. \tag{3.11}$$

Therefore, in this background, we find $F^{(2)}$ is appropriately expanded as

$$F^{(2)} = (\bar{F}_{wy} - Nz\bar{F}_{wx})dw\,dy + \bar{F}_{wx}dw\,dx\,. \tag{3.12}$$

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This reproduces, from another point of view, the expansion of $F^{(2)}$ on a twisted torus in a basis of globally defined 1-forms $\{\eta^x = dx - Nzdy, \eta^y = dy\}$, where we define $\bar{F}_{ab}$ through $F^{(2)} = \bar{F}_{ab}\eta^a \wedge \eta^b$ [11]. The Bianchi identity for $F^{(2)}$ is as usual

$$dF^{(2)} = -N dz\, \bar{F}_{wx} dw\, dy = -\bar{F}_{wx} f^x_{yz} dw\, dy\, dz \tag{3.13}$$

which we may freely integrate over the non-twisted (w, y, z)-cycle to obtain the constraint term (in the absence of other fluxes)

$$\bar{F}_{x[w} f^x_{yz]} = 0\,. \tag{3.14}$$

As the metric (3.9) does not depend on y, we may perform another T-duality in the y direction to arrive at a T^3 with nongeometric flux $Q^{xy}_z = N$. The metric on this background is

$$ds^2 = \frac{1}{1+N^2z^2}\left(dx^2 + dy^2\right) + dz^2 \tag{3.15}$$

and the B-field becomes

$$B_{xy} = \frac{Nz}{1+N^2z^2}\,. \tag{3.16}$$

In this background, the nontrivial monodromy is $\frac{1}{\rho} \to \frac{1}{\rho} + N$ as $z \to z+1$, which mixes the metric and the B-field of the two-torus. Given our particular choice of gauge, the presence of the non-zero Q-flux is described by the $\mathrm{SL}(2,\mathbb{R})$ matrix

$$\begin{pmatrix} 1 & 0 \\ Nz & 1 \end{pmatrix}_\rho\,. \tag{3.17}$$

Now consider the behavior of Ramond-Ramond fields in this background. Returning to type-IIB with the field strengths $\bar{F}_{wxy}$ and $\bar{F}_w$ turned on, we can see that in the presence of the Q-flux, we must write $F^{(1)}$ as

$$F^{(1)} = (\bar{F}_w + Nz\, \bar{F}_{wxy})dw\,, \tag{3.18}$$

and therefore we obtain the modified Bianchi identity

$$dF^{(1)} = -N\bar{F}_{wxy} dw\, dz = -Q^{xy}_z \bar{F}_{xyw} dw\, dz\,. \tag{3.19}$$

Further, as there are no nongeometric twists on the directions z and w, we may regard $dF^{(1)}$ as a two-form without ambiguity. Integrating this two-form over the (z, w) 2-cycle, we find the constraint term (again in the absence of other fluxes)

$$\bar{F}_{xy[w} Q^{xy}_{z]} = 0\,. \tag{3.20}$$

This is an example of a new kind of constraint, which will be important to take into account in constructing nongeometric flux compactifications; we will find the general constraints incorporating this type of term in the next section via T-duality.

There is a natural question that arises at this point: Given a nonzero Q^{xy}_z, can one meaningfully perform another T-duality in the z-direction? By analogy with the above, we

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would expect that such an operation raises another index, producing the object we have denoted R^{xyz}. In section 2, we have argued for the existence of this object in the IIA theory by asserting that the four dimensional superpotential in IIA must agree through T-duality with that of the IIB theory, so it would be nice to be able to see this directly arise from T-dualizing Q_z^{xy}.

The simplest situation in which one would expect the R^{abc} terms to appear is in a T^3 compactification with a single type of flux, where we would hope to have a sequence of T-duality transformations on NS-NS fluxes given by

$$\bar{H}_{xyz} \xleftrightarrow{T_x} f^x_{yz} \xleftrightarrow{T_y} Q_z^{xy} \xleftrightarrow{T_z} R^{xyz}\,. \tag{3.21}$$

As we have discussed above, the first two steps in this procedure can be implemented while thinking of the T^3 as a T^2 bundle over an S^1, but this interpretation breaks down in the final step because of the necessary z-dependence of the metric and B-field describing Q_z^{xy}. We leave the ten-dimensional interpretation of this flux for future work. For practical purposes such as computing flux vacua and the properties of the low-energy effective action, we can certainly make progress without an explicit description of these nongeometric R-fluxes; all we need for such computations is the four-dimensional superpotential which uses these objects. Eventually, however, for a full understanding of vacua with R-fluxes, we need some way of explicitly describing such compactifications in string theory.

It is possible that the R^{xyz} have no interpretation using conventional notions of local spacetime, and in this sense are truly nongeometric. Studying these objects may help us to better understand how both geometric and nongeometric structures may emerge from a fundamental formulation of string theory. Indeed, we can argue that a truly background-independent formulation of string theory (such as string field theory) must include backgrounds with nongeometric R-fluxes as follows: Imagine we have a complete, background-independent formulation of both type-IIA and IIB string theory. The formulation of IIA and IIB in a standard toroidal compactification background with no fluxes must be equivalent. In the IIB background there are SUSY flux vacua with the flux $H_{\alpha\beta\gamma}$ turned on; many explicit examples of such vacua were described in [5, 33]. Such a background should be connected to the background without fluxes in a nonperturbative fashion in the complete formulation of the IIB theory. But the background-independent descriptions of the IIA and IIB theories must be equivalent. Thus, there must be a nonperturbative procedure in the IIA model for going from the vacuum without fluxes to the vacuum with R-flux. This indicates that R-fluxes must have meaning in a background-independent formulation of the theory. Of course, one might prefer the IIB description without R-flux when it is available, but generically there may be vacua where both the IIA and IIB descriptions have R-flux, in which case a more explicit description of the nature of these nongeometric fluxes would be highly desirable.

It is also worth thinking about the worldsheet interpretation of these spaces. Many nongeometric compactifications that have been studied in the past [21 − 23] have been claimed to be related to asymmetric orbifolds [42]. One particular type of compactification features nontrivial twists by an element of the U-duality group when going around a circular direction [21, 23]. On general grounds, it was shown in [21] that in such examples some moduli are fixed by requiring that the vacuum lie at the minimum of a Scherk-Schwarz potential; this is essentially the same as saying that the moduli will always lie at fixed points of the U-duality twist.

It is not, however, necessarily true that the kinds of nongeometric vacua we present in this paper have asymmetric orbifold descriptions. We can see from the T^3 example above that the monodromy $1/\rho \to 1/\rho + N$ has no fixed points, and as such does not fall into the class of vacua considered in [21, 23]. As such, we have no particular reason to believe that the compactifications we consider here generically have asymmetric orbifold descriptions. It may be possible to construct some more general asymmetric CFT in which U-duality twists are incorporated at the level of the world-sheet action. Although the f-, Q-, and R-fluxes are all in the NS-NS sector, and thus may have descriptions with nontrivial boundary conditions implemented on a conventional string worldsheet, this would presumably require developing some new technology to understand fully.

4. Constraints

In this section we discuss constraints on the fluxes. These constraints arise in two closely related ways. First, the new fluxes contribute to the tadpole constraints associated with R-R fields. Second, the new fluxes contribute to the Bianchi identities for the R-R and NS-NS fields. In this section we first derive the general R-R and NS-NS constraints on fluxes using T-duality, and then specialize to the particular toroidal compactification of interest in this paper.

4.1 R-R tadpole and Bianchi identity constraints

In this subsection we discuss the constraints on the fluxes coming from the Ramond-Ramond tadpole conditions and Bianchi identities. The simplest constraint arises in the IIB theory, where there is a tadpole for the R-R four-form field A_4 arising from the Chern-Simons term $\int A_4 \wedge H_3 \wedge F_3$. A_4 is also sourced by local D3-brane and O3-plane contributions. Integrating this tadpole constraint over a six-dimensional compactification manifold gives the topological constraint

$$\bar{F}_{[abc}\bar{H}_{def]} + \text{local} = 0\,. \tag{4.1}$$

By repeated application of T-duality, including both geometric fluxes f^a_{bc} and nongeometric fluxes Q^{ab}_c and R^{abc}, in the absence of local sources we have the R-R constraints

$$\bar{F}_{[abc}\bar{H}_{def]} = 0 \tag{4.2}$$

$$\bar{F}_{x[abc}f^x_{de]} - \bar{F}_{[ab}\bar{H}_{cde]} = 0 \tag{4.3}$$

$$\bar{F}_{xy[abc}Q^{xy}_{d]} - 3\bar{F}_{x[ab}f^x_{cd]} - 2\bar{F}_{[a}\bar{H}_{bcd]} = 0 \tag{4.4}$$

$$\bar{F}_{xyz[abc]}R^{xyz} - 9\bar{F}_{xy[ab}Q^{xy}_{c]} - 18\bar{F}_{x[a}f^x_{bc]} + 6F^{(0)}\bar{H}_{[abc]} = 0 \tag{4.5}$$

$$\bar{F}_{xyz[ab]}R^{xyz} + 6\bar{F}_{xy[a}Q^{xy}_{b]} - 6\bar{F}_x f^x_{[ab]} = 0 \tag{4.6}$$

$$
\begin{aligned}
\bar{F}_{xyza} R^{xyz} - 3\bar{F}_{xy} Q_a^{xy} &= 0 && (4.7)\\
\bar{F}_{xyz} R^{xyz} &= 0\,. && (4.8)
\end{aligned}
$$

If we restrict to the geometric context, these constraints are just versions of the standard Bianchi identity $(d + H)F = 0$. The individual FH, Ff, and FQ terms appearing in these constraints were demonstrated explicitly in the simple example discussed in the previous section. As mentioned previously, in the presence of fixed geometric fluxes, the constraints (4.2)-(4.6) give linear conditions on integrally quantized fluxes $\bar{F}$ which may not be in the cohomology. The extra terms with Q's and R's are additional contributions from nongeometric fluxes. While we have derived these constraints from T-duality on the torus, we expect that there may be a much more general class of compactifications in which these constraints apply.

In our toroidal compactification, we have a set of O3-planes in the IIB model which sets the r.h.s. of (4.2) to $16 \times \binom{6}{3}^{-1}$, where the last factor comes from the combinatorial factors associated with $F \wedge H$. In the IIA model, the corresponding O6-planes set the r.h.s. of (4.5) to 16×6 when the free indices a, b, c run over the directions i, j, k. In terms of the integer coefficients $a_0, \ldots$, the resulting tadpole constraint is the same in both models and is

$$
a_0 b_3 - 3a_1 b_2 + 3a_2 b_1 - a_3 b_0 = 16\,. \tag{4.9}
$$

There is only one further R-R constraint relevant for our model, which comes from (4.6) for the IIB model and again from (4.5) in the IIA model. This constraint becomes

$$
a_0 c_3 + a_1(\check{c}_2 + \hat{c}_2 - \tilde{c}_2) - a_2(\check{c}_1 + \hat{c}_1 - \tilde{c}_1) - a_3 c_0 = 0\,. \tag{4.10}
$$

All remaining tadpole constraints from (4.2)–(4.8) are satisfied automatically in the particular background we are considering here.

4.2 NS-NS Bianchi identity constraints

We can carry out a similar analysis of the NS-NS Bianchi identities from T-duality. In a geometric compactification, the NS-NS fluxes must satisfy

$$
f^x_{[ab} \bar{H}_{cd]x} = 0\,, \tag{4.11}
$$

which comes from the Bianchi identity $dH = 0$. Using T-duality, we find the set of NS-NS constraints

$$
\begin{aligned}
\bar{H}_{x[ab} f^x_{cd]} &= 0 && (4.12)\\
f^a_{x[b} f^x_{cd]} + \bar{H}_{x[bc} Q^{ax}_{d]} &= 0 && (4.13)\\
Q_x^{[ab]} f^x_{[cd]} - 4 f^{[a}_{x[c} Q^{b]x}_{d]} + \bar{H}_{x[cd]} R^{[ab]x} &= 0 && (4.14)\\
Q_x^{[ab} Q_d^{c]x} + f^{[a}_{xd} R^{bc]x} &= 0 && (4.15)\\
Q_x^{[ab} R^{cd]x} &= 0\,. && (4.16)
\end{aligned}
$$

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Finally, in order for the f- and Q-fluxes to be individually T-dual to H-flux, they must satisfy

$$f^x_{xa} = 0 = Q^{ax}_x \,. \tag{4.17}$$

Equations (4.12)–(4.16) have a nice interpretation in the four-dimensional effective theory. Ignoring the R-R fields for the purposes of this discussion, in a reduction on T^6 without flux, the four-dimensional supergravity theory contains a gauge sector with gauge group $U(1)^{12}$ coming from the 10-dimensional metric and B-field. As noted in [12] and developed in [14], adding geometric NS-NS fluxes $\bar{H}_{abc}$ and f^a_{bc} to the compactification makes the gauge algebra of the four-dimensional theory nonabelian; the fluxes appear as structure constants of the gauge algebra. Denoting generators descending from 10-dimensional diffeomorphism invariance as Z_m and generators descending from the 10-dimensional gauge symmetry of B as X^m, the Lie algebra of the compactified theory is [14]

$$[Z_a, Z_b] = \bar{H}_{abc} X^c + f^c_{ab} Z_c \tag{4.18}$$
$$[Z_a, X^b] = -f^b_{ac} X^c \tag{4.19}$$
$$[X^a, X^b] = 0 \,. \tag{4.20}$$

The Jacobi identities of this algebra then yield the purely geometric portion of the NS-NS constraints (4.12)–(4.13). This algebra may be written in a form which is manifestly covariant under the perturbative duality group $O(6,6,\mathbb{Z})$ [14, 18].

By acting on the four-dimensional theory with elements of $O(6,6,\mathbb{Z})$ corresponding to T-duality, we may deduce how to modify this gauge algebra in the presence of nongeometric fluxes Q, R. We find the commutators (4.19)–(4.20) are modified in the obvious way,

$$[Z_a, X^b] = -f^b_{ac} X^c + Q^{bc}_a Z_c \tag{4.21}$$
$$[X^a, X^b] = Q^{ab}_c X^c + R^{abc} Z_c \,. \tag{4.22}$$

The Jacobi identities of this, fully general, algebra now reproduce the full set of NS-NS constraints (4.12)–(4.16).

When applied to our toroidal compactification, the constraints (4.12)–(4.16) lead to a number of conditions on the integer coefficients $b_0, \ldots$. The first set of conditions arises from (4.13) in IIB and yields

$$c_0 b_2 - \tilde{c}_1 b_1 + \hat{c}_1 b_1 - \check{c}_2 b_0 = 0 \tag{4.23}$$
$$\check{c}_1 b_3 - \hat{c}_2 b_2 + \tilde{c}_2 b_2 - c_3 b_1 = 0 \tag{4.24}$$
$$c_0 b_3 - \tilde{c}_1 b_2 + \hat{c}_1 b_2 - \check{c}_2 b_1 = 0 \tag{4.25}$$
$$\check{c}_1 b_2 - \hat{c}_2 b_1 + \tilde{c}_2 b_1 - c_3 b_0 = 0 \,; \tag{4.26}$$

as well as parallel constraints in which all hats and checks are switched through $\hat{c}_i \leftrightarrow \check{c}_i$. In IIA these constraints come from, in order, (4.13), (4.15), and (the last two) (4.14). For instance, we obtain (4.23) in IIB from setting $a = \beta, b = \gamma, c = j$, and $d = k$ in (4.13); the others follow similarly. The second set of conditions arises from (4.15) in IIB and yields

$$c_0 \tilde{c}_2 - \check{c}_1^2 + \tilde{c}_1 \hat{c}_1 - \hat{c}_2 c_0 = 0 \tag{4.27}$$

$$c_3\tilde{c}_1 - \check{c}_2^2 + \tilde{c}_2\hat{c}_2 - \hat{c}_1 c_3 = 0 \tag{4.28}$$
$$c_3 c_0 - \check{c}_2\hat{c}_1 + \tilde{c}_2\check{c}_1 - \hat{c}_1\tilde{c}_2 = 0 \tag{4.29}$$
$$\hat{c}_2\tilde{c}_1 - \tilde{c}_1\check{c}_2 + \check{c}_1\hat{c}_2 - c_0 c_3 = 0\,. \tag{4.30}$$

as well as the parallel constraints with hats and checks switched. In IIA, these constraints again come from (4.13), (4.15), and (the last two) (4.14).

We can simplify these conditions significantly by subtracting each equation from its parallel counterpart with hats and checks switched. From (4.27)–(4.30), we find the conditions

$$c_1\Delta_1 = c_0\Delta_2 \tag{4.31}$$
$$c_3\Delta_1 = c_2\Delta_2 \tag{4.32}$$
$$\tilde{c}_2\Delta_1 = -\tilde{c}_1\Delta_2 \tag{4.33}$$
$$(2\tilde{c}_2 + \hat{c}_2 + \check{c}_2)\Delta_1 = (2\tilde{c}_1 + \hat{c}_1 + \check{c}_1)\Delta_2 \tag{4.34}$$

where

$$c_1 = (\tilde{c}_1 + \hat{c}_1 + \check{c}_1) \tag{4.35}$$
$$c_2 = (\tilde{c}_2 + \hat{c}_2 + \check{c}_2) \tag{4.36}$$
$$\Delta_i = \hat{c}_i - \check{c}_i\,, \qquad i \in \{1,2\}\,. \tag{4.37}$$

Assuming both Δ's are nonzero allows us to rewrite equations (4.27)–(4.30) in terms of the three components of c_1. All 4 equations reduce to the same quadratic

$$3\tilde{c}_1^2 + 3\tilde{c}_1(\hat{c}_1 + \check{c}_1) + \hat{c}_1^2 + \check{c}_1^2 + \hat{c}_1\check{c}_1 = 0\,. \tag{4.38}$$

This equation has no real solution for $\tilde{c}_1$ unless $\hat{c}_1 = \check{c}_1$, so $\hat{c}_1$ and $\check{c}_1$ can be identified. A similar argument demonstrates that, even after setting $\hat{c}_1 = \check{c}_1$, we must have $\hat{c}_2 = \check{c}_2$. Thus, the full set of constraints is just (4.23)–(4.26) and (4.27)–(4.30), with $\hat{c}_i = \check{c}_i$.

The equality $\hat{c}_i = \check{c}_i$ implies a convenient anti-symmetry property of the Q_c^{ab} and f_{bc}^a in our model. Given the equality $Q_k^{i\beta} = Q_k^{\alpha j}$, we may through cyclic permutation of the tori obtain $Q_k^{\alpha j} = Q_i^{\beta k} = -Q_i^{k\beta}$. One may show similarly that Q_c^{ab} is antisymmetric under exchange of any upper and lower index, provided that both indices are of the same kind (Greek or Latin); the same is true for f_{bc}^a. Note, however, that neither Q_c^{ab} nor f_{bc}^a is fully antisymmetric in all three indices, since we are not free to exchange Latin and Greek indices.

Given the simplification $\hat{c}_i = \check{c}_i$, the constraints (4.23)–(4.26) and (4.27)–(4.30) can be simplified further. In particular, (4.23) and (4.26) become equivalent, and (4.24) and (4.25) become equivalent when the constraints on the c's are imposed. Details of the parameterization of solutions to these constraints will be presented along with solutions of the SUSY vacuum equations in [39].

We close this section with a brief discussion of S-duality. The IIB theory is invariant under an S-duality symmetry which exchanges the fluxes F_{abc} and H_{abc} (with a change of sign in one direction), while taking $S \to -1/S$. This has the effect in the superpotential of exchanging the integral flux parameters $a_i \leftrightarrow b_i$. We expect that it should be possible to combine this S-duality transformation with T-duality to get a larger U-duality group under which our 4D theory is invariant. The constraint (4.9) is indeed invariant under S-duality. The remaining constraints, however, provide a puzzle. The equation (4.10) is precisely the sum of the independent bc constraints (4.25) and (4.26) when a and b are exchanged. Thus, the constraints are not obviously incompatible with S-duality, but also do not precisely match. This apparent mismatch in constraints presumably arises from the fact that the Q's actually transform nontrivially under S-duality, since they generically mix the B-field and the metric. Indeed, the mismatch can be seen directly by noting that in the FQ term in (4.6) and the HQ term in (4.13) the free indices (and the number of constraints) do not match. It is clearly crucial to better understand the effects of S-duality on nongeometric fluxes. We leave this as an open question for future work.

5. Conclusions

In this paper we have developed a framework for systematically describing nongeometric NS-NS fluxes in the context of a simple toroidal compactification of type-II string theory. Like R-R fluxes, the geometric and nongeometric NS-NS fluxes act in some sense as p-forms on a canonically chosen space-time, here T^6, and transform under T-duality by adding and removing lower indices through

$$H_{abc} \xleftrightarrow{T_a} f^a_{bc} \xleftrightarrow{T_b} Q^{ab}_c \xleftrightarrow{T_c} R^{abc} \,. \tag{5.1}$$

While we do not have a complete mathematical description of these objects, at least on the torus we can take (5.1) as a definition of how these nongeometric fluxes transform. The f^a_{bc} fluxes correspond to geometrical fluxes defining a "twisted torus" [12, 14, 21]. The Q^{ab}_c fluxes describe compactifications on locally geometric spaces with nongeometric global boundary conditions, such as previously discussed in [21, 23 – 25]. We can explicitly carry out T-duality from $H \to f \to Q$ using standard Buscher T-duality rules, so our discussion here is on well-established ground. We cannot, however, use the Buscher rules to T-dualize $Q^{ab}_c \to R^{abc}$, just as the Buscher rules on R-R potentials $A^{(p)}$ cannot lead to a direct construction of the R-R 0-form flux $F^{(0)}$. In this sense, the last T-duality in (5.1) must at this point be taken as a formal definition.

The need to include nongeometric fluxes of the R-type becomes clear in our construction of the superpotential describing the moduli of the toroidal compactification to four dimensions. We have used concrete T-duality constructions to understand how the Q's extend the superpotential in the type-IIB case, where there are no R's allowed in our particular orientifold compactification due to parity constraints. The consistency of the IIA and IIB pictures then forces us to the conclusion that the nongeometrical fluxes R^{abc} must be included on the IIA side. An important open question is whether these R-type fluxes admit a locally geometric description like the Q-type fluxes.

In this paper we have focused on nongeometric fluxes associated with a toroidal compactification. It is natural to ask how this structure generalizes to other Calabi-Yau manifolds. The structure we have found indicates that nongeometric fluxes may be thought

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of as some additional data which decorates the structure of some particular Calabi-Yau geometry. This could naturally lead to a generalization of mirror symmetry, in which a Calabi-Yau in IIA or IIB decorated with one set of general NS-NS H, geometric, and nongeometric fluxes is mapped through mirror symmetry to the mirror Calabi-Yau in IIB or IIA decorated with the dual set of NS-NS fluxes. In particular, in the picture of mirror symmetry as T-duality on a toroidal fibration [43], H-flux with one leg on the T^3 fiber would map to geometric f-flux, H-flux with two legs on the T^3 fiber would map to nongeometric Q-flux, and H-flux wrapping the T^3 fiber would map to R-flux on the mirror Calabi-Yau. The situation where the mirror of a Calabi-Yau with H-flux is geometrical (i.e., the H-flux has 0 or 1 legs on the T^3) has recently been described in detail in [8, 9], following a suggestion in [44], using the generalized Calabi-Yau geometry developed by Hitchin; it would be very interesting to understand whether there is a precise way of extending that work to the nongeometric context considered here. The generalized tadpole and integrated Bianchi identities we derived in this paper should be valid in a more general context than just the toroidal model considered here, and may provide a good starting point for the concrete generalization of the picture presented in this paper.

It would also be nice to understand how S-duality fits into this framework. As we have discussed at the end of section 4, it is natural to expect that the framework we discuss here should be invariant under a full U-duality group generated by T-duality and S-duality transformations. The constraints we have found on the geometric and nongeometric fluxes seem compatible with S-duality, but are not manifestly invariant, so some additional structure may be needed to form the full U-duality invariant picture. We leave the resolution of this question as an outstanding problem for future work.

In this paper we have focused on a set of essentially topological features of string compactifications characterized by a general set of NS-NS fluxes. We have described the interplay between these integral fluxes and a set of degrees of freedom (the torus moduli) chosen by considering the light degrees of freedom in the particular background without fluxes. As we change fluxes, in different regions of flux space other stringy degrees of freedom will become light, as discussed for example in [45]. Thus, in many cases the low-energy effective theory described by the superpotential we have computed here will not give a complete description of the physics. This is an issue with any classification of flux vacua, but is more acute here where we do not necessarily have tools to assess the validity of the low-energy theory when all nongeometric fluxes are turned on. Indeed, these nongeometric flux compactifications may generically appear at sub-string scales where the supergravity approximation is not valid; since these compactifications also have R-R fluxes, and, even in the locally geometric case, have complicated boundary conditions, we currently have no way of describing these backgrounds precisely using perturbative string theory. It is clearly important to understand better how the compactifications we describe here can be understood in terms of some fundamental formulation of string/M-theory. In the full theory, the fluxes we have described here should be a useful tool for classifying and understanding string backgrounds. In some cases, such as those dual to geometric compactifications in which the low-energy effective description is valid, we know by duality that the low-energy effective description given by the superpotential we have computed in

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terms of nongeometric fluxes will still be valid. It is likely that there are other backgrounds which have no geometric dual, in which this low-energy description is still valid, though these backgrounds will be significantly harder to identify.

An obvious application of the formalism developed in this paper is to classify the full landscape of type-II compactifications on tori with general NS-NS fluxes. The first step in this program would be to determine the vacua arising from the superpotential we have computed here, after which it is necessary to determine corrections to the classical vacuum, including those from other fields which may become light as mentioned above. We have explicitly computed the superpotential for the simplest model with 3 moduli, as well as all constraints on the fluxes. Solutions to this superpotential will include not only all geometric IIA and IIB flux compactifications in this class, but also compactifications which involve nongeometric fluxes either in one or both pictures. A more detailed analysis of the solution space for this model is currently underway and will be reported elsewhere [39]. Unless there is some unexpected general obstruction to the solution of the SUSY equations, it may be possible to use methods developed here to demonstrate conclusively that generic string vacua are nongeometric, increasing yet further the size of the enormous haystack known as the "string landscape", in which we hope to find a compactification correctly describing our world's phenomenology and cosmology.

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